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Brief paper

Modularized design for cooperative control and plug-and-play operation of networked heterogeneous systems☆

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ABSTRACT

In this paper, a cooperative control analysis and design method is investigated for heterogeneous dynamical systems that may be of arbitrary relative degree or nonminimum-phase or both. To achieve consensus or cooperative stability, a negative value of input-feedforward passivity index is used to accommodate and analyze such systems, and the magnitude of the index value is also used as the impact coefficient to quantify the impacts of heterogeneous dynamics of these systems on their networked operations. Physical-system-level designs are explicitly carried out to make individual linear and nonlinear systems (which are either feedback linearizable or nonminimum phase of certain form) become passivity-short and to embed one pure integrator into their input–output dynamics. The network-level distributed control can simply be chosen without any knowledge of the heterogeneous dynamics but with only information of an upper bound on their impact coefficients. It is shown, using the impact equivalence principle, that these controls separately designed but implemented together always ensure either local or global consensus and that a global non-trivial consensus emerges if and only if the information network has at least one globally reachable node or is varying but cumulatively connected. The proposed methodology of fully modularized designs unravels complexity of analyzing and designing cyber–physical systems and enables their plug-and-play into networked operations.

1. Introduction

Cooperative control deals with networked physical systems and network-enabled distributed controls, and its analysis and design involve both dynamics and control of individual physical systems as well as local communication networks and information-structured controls. Until recently, several analysis and design techniques for cooperative control have been developed, and they include the graph-theoretical methods of composite graph connectivity (Jadbabaie, Lin, & Morse, 2003; Lin, Brouchke, & Francis, 2004; Ren & Beard, 2005) and proximity graphs (Cortes, Martinez, & Bullo, 2006), the matrix theoretical approach (Qu, Wang, & Hull, 2008), the Lyapunov-based methods of passivity and circle criterion (Arcak, 2007; Chopra & Spong, 2006; Wu, 2001), set-valued Lyapunov functions (Moreau, 2005), non-smooth analysis and sub-tangentiality conditions (Lin, Francis, & Maggiore, 2007), cooperative control Lyapunov function and topology-based comparison theorems (Qu, 2008, 2009). Their applications to dynamic systems cover a wide range of models which include the simple particle model (Vicsek, Czirok, Jacob, Cohen, & Shochet, 1995), the first-order integrator model (Jadbabaie et al., 2003; Lin et al., 2004; Ren & Beard, 2005) or passive systems (Arcak, 2007), the single integrator model with delay (Fax & Murray, 2004; Saber & Murray, 2004), the linear double integrator model (Tanner, Jadbabaie, & Pappas, 2007), and cooperative canonical systems (Qu et al., 2008). In spite of these advances, there are still many applications of networked control of heterogeneous systems that the current theory and design methods cannot handle, especially when there is a need to enable their plug-and-play into their networked operations. As an example, the analysis and design techniques of cooperative control that involve cyber–physical systems, such as the control and optimization of distributed generation and storage devices in smart grids (see Maknouninejad, Lin, Harno, Qu, & Simaan, 2012 and Xin,

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Qu, Seuss, & Maknouninejad, 2011 and references therein), need to be further developed. In particular, new techniques are desired to properly unwind the entanglement among system dynamics, (unknown) network information flows, and individual as well as network-level control designs. Appropriate impact analysis and design separation will enable us to obtain general conclusions on heterogeneous physical systems and their networked operations.

Often, physical systems have heterogeneous dynamics and their networked operations should be maintained even when some of their physical components get upgraded or exchanged. Accordingly, there is a need to characterize what physical systems are ready for plug-and-play operation and how their controls and network-enabled distributed controls can be designed separately.

The framework of dissipativity provides a way of analyzing input–output properties of certain nonlinear systems, and hence it can also be used to investigate operation of networked systems. It is worth noting that, although the concept of passivity is limited to minimum-phase systems of relative degree one (or zero), it has been widely used in designing model reference adaptive control (Parks, 1966), control of robotic manipulators (Spong & Vidyasagar, 1989), and adaptive control of nonlinear systems (Kokotovic, Krstic, & Kanellakopoulos, 1992). While these controls are to achieve asymptotic stability (i.e., a trivial consensus), a nontrivial consensus is a more general stability concept that captures the emergent behavior of networked dynamic systems, and its value is dependent upon both initial conditions of the systems and network topology. Consensus of passive systems has been shown for balanced and strongly-connected graphs in Chopra and Spong (2006) and for strongly-connected graphs in Zhang, Lewis, and Qu (2012). These two results provide a hint that there are classes of physical systems which are ready for networked operation and for which distributed control can be designed independently of system dynamics.

The objective of this paper is to develop a fully modular design methodology by which a self-feedback control can be designed individually for each of heterogeneous systems while their network-level distributed control can also be synthesized separately. To this end, we use the concept of passivity shortage (for systems whose input-feedforward passivity index value is negative) to include systems of high relative degree and nonminimum phase and to provide the measure of quantifying the impacts of heterogeneous dynamics on their networked operations. Our approach shows that, for dynamical systems to achieve a nontrivial consensus, their individual closed-loop dynamics should be Lyapunov stable but not asymptotically stable, and these individual systems should be networked with positive network connections and individual negative output feedback. We consider modular designs to achieve these properties through appropriate canonical forms as well as all possible structures of information flow, and we show that analysis and design of distributed control does not require explicit knowledge of system dynamics but only needs an upper bounded on their maximum impact. While the preliminary version of so-called generalized passive systems was introduced in Qu (2012), the results reported therein were limited to affine systems, the information topology was confined to be fixed and strongly-connected, and design procedures are not presented. In contrast, this paper develops a fully modular systematic design methodology and untangles the complexity of interactions between system dynamics and network topology.

The remainder of the paper is organized as follows. In Section 2, the problem of modularly designing self-feedback and distributed controls is formulated. In Section 3, tools for analyzing input–output properties of nonlinear systems and their networked operation are developed. In Section 4, two procedures for designing self-feedback controls are detailed to make linear systems and affine nonlinear minimum-phase systems become cooperative PS systems, then a simple design of distributed control is presented, and the so-called impact equivalence principle is established to ensure the emergence of nontrivial consensus under separately designed controls. It is shown in Section 5 that the impact equivalence principle also holds for time-varying information networks. Section 6 contains concluding remarks.

2. Problem formulation

Consider n_s heterogeneous physical systems in the form of

$$\dot{z}_i = \mathcal{F}_i(z_i, v_i), \qquad y_i = H_i(z_i), \tag{1}$$

where $i \in \{1, \ldots, n_s\}$. In (1), $z_i \in \Re^{n_i}$ is the state, $v_i \in \Re^m$ is the control to be designed, and $y_i \in \Re^m$ is the output, of the ith system. The functions $\mathcal{F}_i(\cdot, \cdot)$ and $H_i(\cdot)$ represent system/output dynamics and are of appropriate dimensions. To achieve consensus, or cooperative stability (defined below), all the outputs y_i need to be of the same dimension, but it would be straightforward to consider the case that $v_i \in \Re^{m_i}$ with $m_i \geq m$. It is assumed that the functions are differentiable, $H_i(0) = 0$, and $\partial H(z_i)/\partial z_i$ has rank m.

The cooperative control problem involves designing network-enabled controls for the systems in (1) based on the information structure represented by a digraph $(\mathcal{V}, \mathcal{E}(t))$, where $\mathcal{V}$ denotes the set of n_s nodes and $\mathcal{E}(t)$ denotes the set of directed edges. Equivalently, the local information flow can be characterized by the binary sensing/communication matrix

$$S(t) = \left[S_{ij}(t)\right] \in \Re^{n_s \times n_s}, \qquad S_{ii}(t) \equiv 1, \tag{2}$$

where $S_{ij}(t) = 1$ if $\{j \to i\} \in \mathcal{E}(t)$ (i.e., $(y_i - y_j)$ is available to v_i at the time t), and $S_{ij}(t) = 0$ if otherwise. That is, the presence of edge $\{j \to i\}$ is denoted by binary value of $S_{ij}(t)$.

Definition 1. The system in (1) is said to be cooperative stable if it is Lyapunov stable and if $\lim_{t \to +\infty} y_i(t) = c$ for all i (i.e., $\lim_{t \to +\infty} y(t) = \mathbf{1} \otimes c$, where $\mathbf{1} \in \Re^{n_s}$ is the vector of 1s, $c \in \Re^m$ is the steady state value determined by $z(t_0)$ and by the history of $S(t)$, and $\otimes$ denotes Kronecker product).

A cooperative control is to achieve cooperative stability and, for the systems in (1), it can be chosen to be of form:

$$v_i = v_{s_i}(z_i) + K_i u_i, \tag{3}$$

where $v_{s_i}(\cdot)$ is the *lower-level controller*, K_i is the feedforward gain matrix, and u_i is the *higher-level distributed controller* of form

$$u_i = u_i\left((y_1 - y_i)S_{i1}, \ldots, (y_{n_s} - y_i)S_{in_s}\right). \tag{4}$$

Under control (3), the systems in (1) become individually closed-loop as

$$\dot{z}_i = \mathcal{F}_i(z_i, v_{s_i} + K_i u_i) \triangleq \mathcal{F}_i^c(z_i, u_i), \qquad y_i = H_i(z_i); \tag{5}$$

should the systems be affine,

$$\dot{z}_i = F_i^c(z_i) + G_i(z_i)u_i, \qquad y_i = H_i(z_i). \tag{6}$$

Lower-level control $v_{s_i}(\cdot)$ in (3) is an individual self-feedback control for the ith system to achieve appropriate stability properties for its own dynamics. Higher-level control $u_i(\cdot)$ in (4) is a relative-output feedback control in terms of distributed information from neighboring systems (i.e., $(y_i - y_j)S_{ij}$ rather than state z_j), and it is to ensure cooperative stability under any information network topology with necessary connectivity.

Our goal is to develop the so-called *modularized* designs by which both $v_{s_i}(\cdot)$ and $u_i(\cdot)$ can all be synthesized separately. To this end, input–output measures (in terms of passivity index and output steady state) are presented to quantify the impacts of heterogeneous systems (5) on their networked operation, and the corresponding *impact equivalence principle* is established. Using the principle, network-level distributed control u_i can be designed without the specific knowledge of individual dynamics, and heterogeneous systems of (5) can be put into service anywhere. That is, all the systems satisfying the impact measures are *plug-and-play* ready for their networked operations.

3. Tools for analyzing input–output properties

In this section, several tools are developed to investigate such input–output properties as input-feedforward passivity and output steady state for nonlinear systems and their networked connections. These properties will be used to develop systematic designs for *passivity-short systems* in the subsequent sections.

3.1. Passivity-short (PS) systems

Dissipativity theory (Willems, 1972) has been used to define a number of passivity concepts. Passivity and L_2 gain[2]are the most commonly used forms of dissipativity, and they have been extensively investigated (Brogliato, Lozano, Maschke, & Egeland, 2007; Byrnes, Isidori, & Willems, 1991; Popov, 1973; van der Schaft, 2000; Zhao & Hill, 2008). In addition, more general concepts such as passivity shortage (Sepulchre, Jankovic, & Kokotovic, 1997), passivity indices (McCourt & Antsaklis, 2010) and generalized passivity (Qu, 2012) have been introduced.

Definition 2. The ith system in (5) is said to be dissipative with a storage function $V_i(z_i)$ and a supply rate $\Phi_i(z_i, u_i)$ if $V_i(z_i)$ is positive semi-definite (p.s.d.) and

$$V_i(z_i(t)) - V_i(z_i(0)) \le \int_0^t \Phi_i(z_i(\tau), u_i(\tau))d\tau. \tag{7}$$

The input–output pair $\{u_i, \ y_i\}$ of the ith system in (5) is said to be input feedforward passive if, for some p.s.d. function $\eta_i(\cdot)$,

$$\Phi_i(z_i, u_i) = -\eta_i(z_i) + u_i^T y_i + \frac{\epsilon_i}{2}\|u_i\|^2, \tag{8}$$

where quantity $(-\epsilon_i)$ is called the index of input feedforward passivity. If $\epsilon_i \le 0$ in (8), the index value is nonnegative and the system is said to be passive. If $\epsilon_i \ge 0$, the index value is negative, the system is said to be passivity-short (PS), and ϵ_i in (8) is called impact coefficient.

It is clear from inequality (7) that PS systems include passive systems as special cases. The following three propositions provide several sets of conditions to check whether an input–output pair is PS. Their proofs are included in the Appendix.

Proposition 1. *Consider the ith system in (5).*

(i) *Its pair $\{u_i, \ y_i\}$ is PS if and only if the fictitious pair $\{u_i, y_i^a\}$ is passive, where $y_i^a = y_i + 0.5\epsilon_i u_i$ is the augmented output.*
(ii) *Its pair $\{u_i, \ y_i\}$ is PS if, for a $\mathcal{C}^1$ storage function $V_i(\cdot)$ and for some constants $\gamma_{f_i}, \gamma_{h_i}, \gamma_{i3}, \gamma_{i4} > 0$,*

$$\|\mathcal{F}_i^c(z_i, u_i) - \mathcal{F}_i^c(z_i, 0)\| \le \gamma_{f_i}\|u\|, \qquad \|H_i(z_i)\| \le \gamma_{h_i}\|z_i\|, \tag{9}$$

$$\left(\frac{\partial V_i}{\partial z_i}\right)^T \mathcal{F}_i^c(z_i, 0) \le -\gamma_{i3}\|z_i\|^2, \qquad \left\|\frac{\partial V_i}{\partial z_i}\right\| \le \gamma_{i4}\|z_i\|. \tag{10}$$

Part (ii) of Proposition 1 provides a Lyapunov test for nonaffine systems, and part (i) can be used together with existing passivity tests, for instance, Kalman–Yakubovich–Popov (KYP) lemma (Khalil, 2003; Popov, 1973; Wen, 1988) for linear systems. It is worth noting that, by converse theorem (Khalil, 2003), the inequalities in (10) are ensured if zero-command system $\dot{z}_i = \mathcal{F}_i^c(z_i, 0)$ is both exponentially stable and globally Lipschitz but, as shown by the Teel–Hespanha example (Teel & Hespanha, 2004), exponential stability by itself does not imply (10).

[2] L_2 gain of $\gamma \in \Re$ is defined by $\Phi_i(z_i, u_i) = \frac{\gamma^2}{2}\|u_i\|^2 - \frac{1}{2}\|y_i\|^2$, where $\eta(t) \in L_2$ if $\|\eta(t)\|_{L_2} \triangleq \left(\int_{t_0}^{\infty}\|\eta(t)\|^2 dt\right)^{1/2} < \infty$. Similarly, $\eta(t) \in L_\infty$ if $\|\eta(t)\|_{L_\infty} \triangleq \sup_{t \ge t_0}\|\eta(t)\| < \infty$.

Proposition 2. *Suppose that the ith system in (6) has a $\mathcal{C}^1$ storage function. Then, its pair $\{u_i, \ y_i\}$ is PS if and only if $-\mathcal{L}_{F_i^c}V_i$ is p.s.d. and, for some $\epsilon_i > 0$ and $\epsilon_i' \in [0, 1]$,*

$$\eta_i(z_i) \triangleq -\mathcal{L}_{F_i^c}V_i - \frac{1}{2\epsilon_i}\|\mathcal{L}_{G_i}V_i - H_i^T\|^2 \ge -\epsilon_i'\mathcal{L}_{F_i^c}V_i. \tag{11}$$

Proposition 2 is applicable to affine systems and, in light of the KYP property in Byrnes et al. (1991), inequality (11) can be referred to as the *passivity-short KYP property* or, if $\epsilon' > 0$, the *strictly passivity-short KYP property*.

It is straightforward to show using either Proposition 2 or 1 that all linear Hurwitz systems are PS. Nonetheless, PS systems may not necessarily be asymptotically stable, as evidenced by the following proposition.

Proposition 3. *Consider the linear system*

$$\begin{bmatrix}\dot{z}_{i1}\\ \dot{z}_{i2}\end{bmatrix} = \begin{bmatrix}F_{i,11} & 0\\ 0 & 0\end{bmatrix} z_i + \begin{bmatrix}G_{i1}\\ I\end{bmatrix} u_i, \qquad y_i = \begin{bmatrix}H_{i1} & I\end{bmatrix} z_i. \tag{12}$$

It is PS if $F_{i,11}$ is Hurwitz.

It is well known (Khalil, 2003) that a passive system must be minimum-phase and have relative degree of zero or one (or negative one). In comparison, PS systems include those that are of higher relative degrees and/or nonminimum phase, which will be illustrated later by examples (specifically, the first system in Examples 1, and 4). How to design $v_{s_i}(\cdot)$ to make systems become PS is the subject of Section 4.1.

3.2. Positive network interconnection of PS systems

The fundamental property of passive systems is that a *negative-feedback* connection of two passive systems is also passive (Khalil, 2003). The following example illustrates that a negative feedback connection of two PS systems may not be PS. This is because the overall system may become Lyapunov unstable, while by Definition 2 shortage of passivity with a positive definite (p.d.) storage function still yields Lyapunov stability under $u_i = 0$.

Example 1. Consider the two systems and their outputs:

$$\begin{cases}\dot{z}_{11} = z_{12}\\ \dot{z}_{12} = -\frac{1}{2}z_{12} - \frac{1}{2}z_{11} + u_1,\end{cases} \qquad \dot{z}_{21} = u_2, \qquad \begin{cases} y_1 = z_{11}\\ y_2 = z_{21}.\end{cases}$$

The first system is Hurwitz, and pair $\{u_1, y_1\}$ is PS (but not passive due to relative degree 2). Pair $\{u_2, y_2\}$ is passive (and also PS). The negative feedback connection of these two systems is shown in Fig. 1 and described by

$$\frac{d}{dt}\begin{bmatrix}z_{11}\\ z_{12}\\ z_{21}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ -\frac{1}{2} & -\frac{1}{2} & -1\\ 1 & 0 & 0\end{bmatrix}\begin{bmatrix}z_{11}\\ z_{12}\\ z_{21}\end{bmatrix} + \begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix},$$

which is Lyapunov unstable and hence cannot be PS. ◇

PS systems do have the nice property that *positive-feedback* interconnections of PS systems individually with negative output self-feedback are Lyapunov stable. Suppose that matrix S in (2) is constant, the positive-feedback interconnections (with individual negative self-feedbacks) are specified by

$$u_i = k_{y_i}\sum_{j=1}^{n_s}(y_j - y_i)S_{ij} \Longleftrightarrow u = -((K_h L) \otimes I_m)y, \tag{13}$$

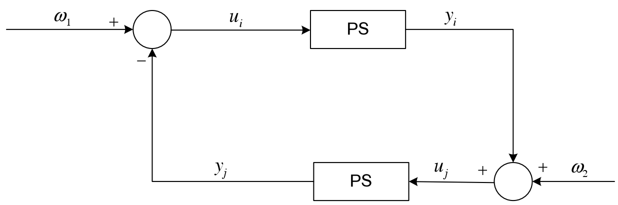

Fig. 1. Negative feedback connection of two PS systems.

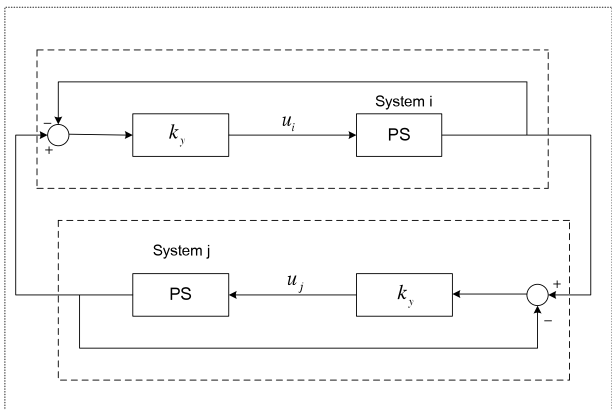

Fig. 2. Positive feedback connection of two PS systems with self negative output feedbacks.

where $L = (D - S)$ is the *Laplacian*, $D = \text{diag}\{S\mathbf{1}\}$ is a diagonal matrix containing the in-degrees of L, and $K_h = \text{diag}\{k_{y_i}\} \in \Re^{n_s \times n_s}$ is the diagonal positive-gain matrix to be chosen. The following lemma establishes cooperative stability for the case if digraph $(\mathcal{V}, \mathcal{E})$ is *strongly connected* (in the sense that every node can be reached from any other node through directed edges) or, equivalently, that matrix S or L is *irreducible* (i.e., $(I + L)^{n_s - 1}$ or $S^{n_s - 1}$ is positive). Note that the left eigenvector and eigenvalues associated with Laplacian L can be estimated distributively (Qu, Li, & Lewis, 2014). Fig. 2 depicts all the connections when $n_s = 2$.

Lemma 1. *Suppose that the systems in* (5) *for* $i = 1, \ldots, n_s$ *are PS with p.d. and radially-unbounded storage functions* $V_i(\cdot)$ *and with impact coefficients* $\epsilon_i \in [0, \overline{\epsilon}]$. *If* L *is irreducible, the distributed control in* (13) *ensures cooperative stability as well as* $(y_i - y_j) \in L_2$ *and* $u \in L_2$ *provided that* $k_{y_i} \in (0, \overline{k}_y)$, *where* $\overline{k}_y = \lambda_2(\Gamma L + L^T \Gamma)/[\overline{\epsilon}\lambda_{\max}(L^T \Gamma L)] > 0$, $\gamma = \text{vec}\{\gamma_i\}$ *is the first left eigenvector of* L *(defined by* $\gamma^T L = 0$*)*, $\Gamma = \text{diag}\{\gamma_i\}$, $\lambda_{\max}(A)$ *denotes the maximum eigenvalue of matrix A, and* $\lambda_2(A)$ *represents the second smallest eigenvalue of matrix A.*

Proof. By Perron–Frobenius theorem, the first left eigenvector γ of irreducible Laplacian L is positive. Define the overall storage function to be $\overline{V}(z) \triangleq \sum_{i=1}^{n_s} \gamma_i k_{y_i}^{-1} V_i(z_i)$. It follows from Definition 2 and (13) that

$$\begin{aligned}\overline{V} &\le \overline{V}(z(0)) + \sum_i \gamma_i k_{y_i}^{-1} \int_0^t \left[-\eta_i + u_i^T y_i + \frac{\epsilon_i}{2}\|u_i\|^2\right] ds \\ &\le \overline{V}(z(0)) - \sum_i \gamma_i k_{y_i}^{-1} \int_0^t \eta_i ds - \frac{1}{2}\int_0^t y^T Q y ds, \end{aligned} \quad (14)$$

where $Q = Q_h \otimes I_m$, $Q_h = W_h - \overline{\epsilon} L^T K_h \Gamma L$, and $W_h = \Gamma L + L^T \Gamma$. It follows from Theorem 4.31 in Qu (2009) that matrix $W_h \in \Re^{n_s \times n_s}$ is p.s.d., is of rank $(n_s - 1)$, and has the property that $\xi^T W_h \xi = 0$ if and only if $\xi = c\mathbf{1}_{n_s}$ for some $c \in \Re$. On the other hand, $L\mathbf{1}_{n_s} = 0$ implies $\mathbf{1}_{n_s}^T Q_h \mathbf{1}_{n_s} = 0$. Since W_h is independent of k_{y_i} while the other matrix product in Q_h is linear in k_{y_i}, we know that, under the choices of $k_{y_i} \in (0, \overline{k}_y)$, term $y^T Q y$ assumes positive values except for being zero when $y_i = y_j$ for all i, j. Hence, Lyapunov stability, $(y_i - y_j) \in L_2$ and $u \in L_2$ can be concluded from (14). It follows from (5) and Lyapunov stability that y_i are uniformly continuous and hence, by Barbalat's lemma (Khalil, 2003), $\lim_{t\to+\infty} y_i(t) = c$ for all i. □

Lemma 1 presents a simple result of cooperative stability but has two limitations. First, it is limited to the case when the digraph is strongly connected. Further analysis is needed for the more general case when the graph is reducible. Second, while $\lim_{t\to+\infty} y_i(t) = c$ is shown, little information is known about c. In the cooperative control problem, the trivial solution of $\lim_{t\to+\infty} y_i(t) = 0$ is of little interest since convergence to the origin can easily be achieved by asymptotically stabilizing every one of the systems without any network-level control. These issues are investigated in the subsequent subsection.

3.3. Nontrivial consensus of PS systems

To study the consensus values, define the equilibrium set Ω_i^e of the ith system in (6) and its output image Ω_i^y as

$$\begin{cases} \Omega_i^e \triangleq \{z_i^e \in \Re^{n_i} : \ F_i^c(z_i^e) = 0\} \\ \Omega_i^y \triangleq \{y_i \in \Re^m : \ y_i = H_i(z_i^e), \ z_i^e \in \Omega_i^e\}. \end{cases} \quad (15)$$

Network-level distributed controls (4) such as (13) are chosen such that, when output consensus is reached, $u_i = 0$. Hence, it follows from (14) and Definition 2 that, should $\eta_i(z_i)$ be p.d., $z_i(t)$ converges to zero and in turn so does $y_i(t)$. That is, in order to achieve a nontrivial consensus, it is necessary that none of the systems in (5) is asymptotically stable under $u_i = 0$. Explicit rank conditions to ensure nontrivial consensus values are stated in the following lemma.

Lemma 2. *Suppose that the ith system in* (6) *satisfies the passivity-short KYP property of* (11). *Then, the equilibrium set* Ω_i^e *and its output image* Ω_i^y *have dimensions of* $\dim(\Omega_i^e) = m$ *and* $\dim(\Omega_i^y) = m$ *provided that the following rank conditions hold (locally) at* $z_i = z_i^e$:

$$\text{rank}\begin{bmatrix} \dfrac{\partial^2 \mathcal{L}_{F_i^c} V_i}{\partial z_i^2} \end{bmatrix} = n_i - m, \qquad \text{rank}\begin{bmatrix} \dfrac{\partial^2 \mathcal{L}_{F_i^c} V_i}{\partial z_i^2} \\ \dfrac{\partial H_i(z_i)}{\partial z_i} \end{bmatrix} = n_i. \quad (16)$$

Proof. We know from Proposition 2 that $\mathcal{L}_{F_i^c} V_i$ is n.s.d. and hence Ω_i^e in (15) is nonempty. For any $z_i^e \in \Omega^e$ and under $u_i = 0$, we have $\dot{V}_i = \mathcal{L}_{F_i^c} V_i = 0$. Hence, function $\mathcal{L}_{F_i^c} V_i$ reaches its maximum at z_i^e, and the steady state of the ith system in (6) is the solution to the algebraic equations:

$$\frac{\partial \mathcal{L}_{F_i^c} V_i(z_i^e)}{\partial z_i^e} = 0, \qquad H(z_i^e) = c. \quad (17)$$

For any arbitrary choice of $c \in \Re^m$, the second equation is always solvable since $\partial H(z_i)/\partial z_i$ is of rank m. For $\dim(\Omega_i^y) = m$, it is necessary that $\dim(\Omega_i^e) \ge m$. The proof is completed by applying the implicit function theorem (see Khalil, 2003) to the above equations. □

For the stabilization problem, singleton $\Omega_i^e = \{0\}$ would be needed; for the problems of output tracking or consensus, $\Omega_i^y = \Re^m$ would be desired. The above lemma provides the technical conditions, and the subsequent example illustrates the corresponding Lyapunov design.

Example 2. Consider the uncertain system

$$\dot{z}_{i1} = z_{i1}^3 + z_{i2}, \qquad \dot{z}_{i2} = \varphi_i(z_i)\theta_i + v_i, \qquad y_i = z_{i1},$$

where θ_i is a vector of unknown parameters. Applying the backstepping adaptive design (Krstic, Kanellakopoulos, & Kokotovic, 1995), we can choose an augmented storage function $V_i'(z_{i1}, z_{i2}, \hat{\theta}_i)$ as

$$V_i' = \frac{k_i}{2}|y_i|^2 + \frac{k_i}{2}\left|z_{i1} + \frac{1}{k_i}(z_{i2} + z_{i1}^3)\right|^2 + \frac{k_i}{2}\|\theta_i - \hat{\theta}_i\|^2$$

and an adaptive feedback control $v_i = -\varphi_i(z_i)\hat{\theta}_i - (2k_i + 3z_{i1}^2)(z_{i1}^3 + z_{i2}) + u_i$ and $\dot{\hat{\theta}}_i = \varphi_i^T(z_i)\left[z_{i1} + \frac{1}{k_i}(z_{i2} + z_{i1}^3)\right]$, under which $\dot{V}_i' = -|z_{i2} + z_{i1}^3|^2 + \frac{1}{k_i}(z_{i2} + z_{i1}^3)u + y_i u_i$. It is apparent that the system satisfies both (11) (with $\epsilon_i \geq 1/(2k_i^2)$) and (16). $\diamond$

The existence of a non-trivial equilibrium set does not necessarily mean that any nontrivial equilibrium value is a asymptotically stable equilibrium. The following theorem ensures both input–output convergence and internal stability so that technical development in the rest of the paper can be done in terms of only input–output properties.

Theorem 1. *Consider the systems in (6) under control*

$$u_i = k_{y_i}[r_i(t) - y_i], \tag{18}$$

where $r_i(t)$ is the command signal satisfying $[r_i - c] \in L_2$ and $\lim_{t\to+\infty} r_i(t) = c$ for some constant $c \in \Re^m$. Then,

- *If the system satisfies the strictly passivity-short KYP property of (11) with a $\mathcal{C}^3$, p.d., radially unbounded storage function V_i and with impact coefficient ϵ_i and if rank condition (16) holds in $\Re^{n_i}$, control (18) with $k_{y_i} \in (0,\ 2/\epsilon_i)$ ensures input-to-state stability (ISS).*
- *Alternatively, if pair $\{u_i, (y_i - c)\}$ of the system is PS with p.d. radially unbounded storage function $V_i^c(z_i, c)$ and impact coefficient $\epsilon_i^c \in [0, 2/k_{y_i})$, the system has Lyapunov stability and unity DC gain (in the sense of $\lim_{t\to+\infty} y_i(t) = c$), and $(y_i - c), u_i \in L_2$.*

Proof. Define a scalar function $\xi(\lambda) \triangleq \mathcal{L}_{F_i^c(w)} V_i(w)|_{w=\lambda z_i}$, where $\lambda \in [0, 1]$. Direct computation yields that $\xi(0) = \left.\frac{\partial \xi(\lambda)}{\partial \lambda}\right|_{\lambda=0} = 0$, $\xi(1) = \mathcal{L}_{F_i^c} V_i$, and

$$\frac{\partial^2 \xi(\lambda)}{\partial \lambda^2} = z_i^T \left.\frac{\partial^2 \mathcal{L}_{F_i^c(w)} V_i(w)}{\partial w^2}\right|_{w=\lambda z_i} z_i. \tag{19}$$

It follows from the mean value theorem in Khalil (2003) that

$$\mathcal{L}_{F_i^c} V_i = \left.\frac{\partial \xi(\lambda)}{\partial \lambda}\right|_{\lambda=\lambda^* \in (0,1)} = \lambda^* \left.\frac{\partial^2 \xi(\lambda)}{\partial \lambda^2}\right|_{\lambda=\lambda^{**} \in (0,\lambda^*)},$$

which together with (19) and rank condition (16) imply that $-\mathcal{L}_{F_i^c} V_i$ is p.s.d. and radially unbounded with respect to all the variables in z_i except for those m variables solved from $y_i = H_i(z_i)$. Hence, the term $(-a_1 \mathcal{L}_{F_i^c} V_i + a_2\|y_i\|^2)$ is p.d. and radially unbounded for any $a_1, a_2 > 0$.

It follows from Proposition 2 and from the strictly passivity-short KYP property that, under control (18),

$$\begin{aligned}\dot{V}_i &\leq -\eta_i(z_i) + k_{y_i} y_i^T(r_i - y_i) + \frac{\epsilon_i}{2}k_{y_i}^2\|r_i - y_i\|^2 \\ &\leq \epsilon_i' \mathcal{L}_{F_i^c} V_i - \frac{1}{2}(2 - \epsilon_i k_y)k_{y_i}\|y_i\|^2 + (1 - \epsilon_i k_y)k_{y_i} r_i^T y_i \\ &\quad + \frac{\epsilon_i}{2}k_{y_i}^2\|r_i\|^2 \end{aligned} \tag{20}$$

$$\begin{aligned} &\leq \epsilon_i' \mathcal{L}_{F_i^c} V_i - \frac{2 - \epsilon_i k_y}{4}k_{y_i}\|y_i\|^2 + \frac{(1 - \epsilon_i k_y)^2}{(2 - \epsilon_i k_y)}k_{y_i}\|r_i\|^2 \\ &\quad + \frac{\epsilon_i}{2}k_{y_i}^2\|r_i\|^2, \end{aligned} \tag{21}$$

from which ISS can be concluded by recalling that the sum of the first two terms on the right hand side of (21) is negative definite and radially unbounded with respect to z_i and by applying Theorem 4.19 in Khalil (2003).

To show both unity DC gain and Lyapunov stability, we let $V_i^c(z_i, c)$ denote the storage function for pair $\{u_i, (y_i - c)\}$. It follows from the pair being PS that, for some $\eta_i^c(\cdot) \geq 0$ and for any $\epsilon' > 0$,

$$\begin{aligned} &V_i^c(z_i, c) - V_i^c(z_i(0), c) + \int_0^t \eta_i^c(z_i)ds \\ &\leq k_{y_i}\int_0^t (r_i - y_i)^T(y_i - c)ds + k_{y_i}^2\frac{\epsilon_i^c}{2}\int_0^t \|r_i - y_i\|^2 ds \\ &\leq -\frac{k_{y_i}[2 - k_{y_i}(\epsilon_i^c + \epsilon')]}{2}\|r_i - y_i\|_{L_2} + \frac{1}{2\epsilon'}\|r_i - c\|_{L_2}, \end{aligned} \tag{22}$$

from which Lyapunov stability and $(y_i - c) \in L_2$ can be concluded by using the Chebyshev inequality $\|y_i - c\|^2 \leq 2(\|y_i - r_i\|^2 + \|r_i - c\|^2)$. Barbalat lemma (Khalil, 2003) can then be invoked to conclude the unity DC gain. $\square$

It is possible to have the output track the input while some of the internal state variables do not settle down. In particular, it is necessary to establish internal stability because a counter example could be constructed according to the equality version of (20) in the above proof. Also in Theorem 1, unity DC gain is established under the condition of pair $\{u_i, (y_i - c)\}$ being PS for any $c \in \Re^m$, which is necessary for achieving nontrivial consensus. The connection from pair $\{u_i, y_i\}$ being PS to pair $\{u_i, (y_i - c)\}$ being PS will be established in Section 3.4.

The following theorem extends Lemma 1 to the leader–followers problem in which all the outputs of the systems in a strongly connected network converge to the steady state of leader $r_0(t)$.

Theorem 2. *Let $r_0(t) \in \Re^m$ with $\lim_{t\to+\infty} r_0(t) = c$ and $(r_0 - c) \in L_2$ denote the state of the leader, and let binary function S_{i0}' represent the connectivity from the leader to the ith system. Consider the systems in (5), and suppose that their pairs $\{u_i, (y_i - c)\}$ are PS with p.d. radially-unbounded storage functions $V_i^c(z_i, c)$ and impact coefficients $\epsilon_i^c \in [0, \overline{\epsilon}]$. Then, if $L' \triangleq \mathrm{vec}\{-S_{i0}'\} \neq 0$, there is a diagonal p.d. matrix Γ_a such that $(\Gamma_a L_a + L_a^T \Gamma_a)$ is p.d., and the following distributed leader–followers control ensures cooperative stability (with $\lim_{t\to+\infty} y_i(t) = c$ for all i), $(y_i - y_j) \in L_2$ and $u \in L_2$:*

$$\begin{aligned} u_i &= k_{y_i}\sum_{j=1}^{n_s}(y_j - y_i)S_{ij} + k_{y_i}(r_0 - y_i)S_{i0}' \\ \Longleftrightarrow u &= -((K_h L_a) \otimes I_m)y - ((K_h L') \otimes I_m)r_0, \end{aligned} \tag{23}$$

where $S = [S_{ij}]$ is as defined in (2), K_h and L are as defined in (13) with L being irreducible, $L_a = L + \mathrm{diag}\{S_{i0}'\}$, and $\overline{L} = [L' \ \ L_a]$ is the overall Laplacian, and $k_{y_i} < \lambda_{\min}(\Gamma_2 L_2 + L_2^T \Gamma_2)/[\overline{\epsilon}\lambda_{\max}(L_2^T \Gamma_2 L_2)]$ with $\lambda_{\min}(A)$ denoting the minimum eigenvalue of matrix A.

Proof. Since $\overline{L}$ has zero row sums and L' is nonpositive and nonzero, it follows from Lemma 4.32 and Theorem 4.25 in Qu (2009) that matrix L_a must be a nonsingular M-matrix and hence diagonal p.d. matrix $\Gamma_a = \mathrm{diag}\{\gamma_i^a\}$ exists such that $W_a = \Gamma_a L_a + L_a^T \Gamma_a$ is p.d. Hence, matrix $Q_a \triangleq W_a - \overline{\epsilon} L_a^T K_h \Gamma_a L_a$ is also p.d. for all small values of k_{y_i} as specified.

It follows from (23) that

$$u = -((K_h L_a) \otimes I_m)(y - c) - ((K_h L') \otimes I_m)(r_0 - c).$$

Define the overall storage function to be $\overline{V}^c(z,c) \triangleq \sum_{i=1}^{n_s} \gamma_i^a k_{y_i}^{-1} V_i^c(z_i,c)$. In the case that $r_0(t) = c$ and for some $\eta_i^c(\cdot) \geq 0$ and $\epsilon_i^c \in [0, \overline{\epsilon}]$, we have

$$\begin{aligned}
&\overline{V}^c(z,c) - \overline{V}^c(z(0),c) \\
&\leq \sum_i \frac{\gamma_i}{k_{y_i}} \int_0^t \left[-\eta_i^c + u_i^T(y_i - c) + \frac{\epsilon_i^c}{2}\|u_i\|^2 \right] ds \\
&\leq -\frac{1}{2}\int_0^t \left[\sum_i \frac{2\gamma_i \eta_i^c}{k_{y_i}} + (y-c)^T \overline{Q}(y-c) \right] ds,
\end{aligned} \tag{24}$$

where $\overline{Q} = Q_a \otimes I_m$. For the general case of $(r_0 - c) \in L_2$, we can establish (as we did with inequality (22)) inequality (24) by including an additive term of $\|r_0 - c\|_{L_2}$ into the right hand side. Hence, cooperative stability is concluded. □

3.4. Canonical form of cooperative PS systems

In this subsection, a canonical form is developed for PS systems that can achieve nontrivial consensus. It follows from rank condition (16) and the proof of Lemma 2 that, for PS systems (of pairs $\{u_i, y_i\}$) to be able to reach a non-trivial consensus anywhere in $\Re^m$ (i.e., $\Omega_i^y = \Re^m$), each of the systems must implicitly contain m pure integrators (because m of the equations in (17) become degenerate). As illustrated by Theorems 1 and 2 and their proofs, cooperative stability (of $\lim_{t\to+\infty} y_i(t) = c$) could be easily established if pairs $\{u_i, (y_i - c)\}$ are known to be PS for all $c \in \Re^m$, while a Lyapunov argument based on the rank condition is technically involved (in terms of such requirement as differentiability, strictly passivity-short KYP condition, etc.). Nonetheless, the existence of pure integrators allows us to choose the canonical form of desired PS systems as stated below and revealed by the rank condition.

Definition 3. The ith system in (6) is said to be a cooperative PS system if there exists a diffeomorphic transformation $z_i = \mathcal{Z}_i(w_i)$ under which the system in (6) is transformed into the following canonical form:

$$\begin{aligned}
&\begin{bmatrix} \dot{w}_{i1} \\ \dot{w}_{i2} \end{bmatrix} = \begin{bmatrix} F_{w_{i1}}(w_{i1}, w_{i2}) \\ 0 \end{bmatrix} + \begin{bmatrix} G_{w_{i1}}(w_{i1}, w_{i2}) \\ I \end{bmatrix} u_i, \\
&y_i = H_{w_i}(w_{i1}, w_{i2}) + \beta_i w_{i2},
\end{aligned} \tag{25}$$

where $H_{w_i}(0, w_{i2}) = 0$ and constant $\beta_i \neq 0$ has the properties that, for any $u_i \in L_2$ and for uniformly bounded w_{i2}, the reduced order system $\dot{w}_{i1} = F_{w_{i1}}(w_{i1}, w_{i2}) + G_{w_{i1}}(w_{i1}, w_{i2})u_i$ is globally asymptotically stable so that $H_{w_i}(\cdot,\cdot) \in L_2$ (or uniformly bounded if $H_{w_i}(\cdot,\cdot) \equiv 0$).

The ith system in (25) is PS and has the equilibria of $w_i^e = [0 \ c^T]^T$ for any $c \in \Re^m$, it satisfies the rank condition (under differentiability), and it includes the system in (12) as a special case. Since Lyapunov criteria exist to check L_2 stability of its reduced order system (see Theorems 5.1 and 5.5 in Khalil, 2003), the following lemma focuses upon the relationship of PS property between pairs $\{u_i, y_i\}$ and $\{u_i, (y_i - c)\}$. The relationship together with Theorem 1 makes the modularized design possible. The corresponding state transformation is illustrated in Example 3.

Lemma 3. *If pair $\{u_i, y_i\}$ of system (25) is PS with a $\mathcal{C}^1$ storage function of the form*

$$V_{w_i}(w_i) = V_{w_{i1}}(w_{i1}, w_{i2}) + \frac{1}{2}\beta_i\|w_{i2}\|^2, \tag{26}$$

then its pair $\{u_i, (y_i - c)\}$ is also PS with the same impact coefficient and for all $c \in \Re^m$.

Proof. It follows from $\{u_i, y_i\}$ being PS that

$$\begin{aligned}
\dot{V}_{w_i}(w_i) &= [F_{w_{i1}} + G_{w_{i1}}u_i]^T \frac{\partial V_{w_{i1}}}{\partial w_{i1}} + u_i^T \frac{\partial V_{w_{i1}}}{\partial w_{i2}} + \beta_i u_i^T w_{i2} \\
&\leq -\eta_{w_i}(w_{i1}) + u_i^T y_i + \frac{\epsilon_i}{2}\|u_i\|^2,
\end{aligned} \tag{27}$$

where $\epsilon_i \geq 0$. Now, consider storage function $V_{w_i}^c(w_i, c) = V_{w_{i1}}(w_{i1}, w_{i2}) + \frac{1}{2}\beta_i \left\| w_{i2} - \frac{c}{\beta_i} \right\|^2$, which is p.s.d. It follows from (27) that $\dot{V}_{w_i}^c(w_i, c) \leq -\eta_{w_i}(w_{i1}) + u_i^T(y_i - c) + \frac{\epsilon_i}{2}\|u_i\|^2$, from which pair $\{u_i, (y_i - c)\}$ being PS is obvious. □

Example 3. Consider the following system:

$$\begin{cases} \dot{z}_{i1} = z_{i1}^3 + z_{i2} \\ \dot{z}_{i2} = -(2k_i + 3z_{i1}^2)(z_{i1}^3 + z_{i2}) + u_i, \end{cases} \qquad y_i = z_{i1}.$$

As was shown in Example 2, the pair $\{u_i, \ y_i\}$ is PS. Under state transformation of $w_{i1} = z_{i1}^3 + z_{i2}$ and $w_{i2} = 2k_i z_{i1} + (z_{i1}^3 + z_{i2})$, the system is transformed into (25) as

$$\dot{w}_{i1} = -2k_i w_{i1} + u_i, \qquad \dot{w}_{i2} = u_i, \qquad y_i = -\frac{1}{2k_i}w_{i1} + \frac{1}{2k_i}w_{i2},$$

in which w_{i1} has L_2 stability. ◇

4. Modularized design: fixed topology

The proposed modularized design methodology is to separately synthesize self-feedback controls for each of the individual systems as well as distributed controls.

4.1. Lower-level designs of self-feedback control

At the lower level, self-feedback control $v_{s_i}(z_i)$ should individually be chosen to make every physical system of form (6) become a cooperative PS system as defined in Definition 3. Two systematic design procedures are presented below: one for linear systems, and the other for feedback linearizable nonlinear systems.

The first procedure is a matrix-theoretical design approach applicable to all linear systems of form

$$\dot{z}_i = F_i z_i + G_i v_i, \qquad y_i = H_i z_i, \tag{28}$$

where $z_i \in \Re^{n_i}$, $\{F_i, \ G_i, \ H_i\}$ is an irreducible realization (Chen, 1984) of the input–output mapping from v_i to y_i, and $G_i = \begin{bmatrix} G_{i1} & \cdots & G_{im} \end{bmatrix}$. For the system in (28), the self-feedback control[3] is of form

$$v_i = -K_{s_i} z_i + K_i u_i, \tag{29}$$

where K_{s_i} is the self-feedback gain matrix, and K_i is the feedforward gain matrix. Given controllability, positive integers p_{ij} exist such that controllability matrix

$$\begin{cases} \mathfrak{C}_i = \left[G_{i1} \ \cdots \ F_i^{p_{i1}-1}G_{i1} \middle| \cdots \middle| G_{im} \ \cdots \ F_i^{p_{im}-1}G_{im} \right] \\ \displaystyle\sum_{j=1}^m p_{ij} = n_i \end{cases} \tag{30}$$

is nonsingular. The following theorem provides the design of gain matrices K_{s_i} and K_i.

[3] If z_i is not available, a dynamic self-feedback control v_i can be designed by using the separation principle and employing an observer to estimate z_i from y_i. Should the realization in (28) not be minimal, an irreducible realization can be found (Chen, 1984) before applying the proposed procedure. In the event that system (28) with $K_{s_i} = 0$ can be mapped into (12), v_i does not require absolute measurement of any state variable.

Theorem 3. *For any chosen set of stable eigenvalues λ_{ij} ($j = 1, \ldots, (n_i - m)$), define desired characteristic polynomials $E_{ij}(s)$ to be $E_{ij}(s) = sE'_{ij}(s)$, where*

$$\prod_{j=1}^{m} E'_{ij}(s) = (s - \lambda_{i1}) \cdots (s - \lambda_{i(n_i-m)}). \tag{31}$$

Let q_{ij} denote the $(p_{i1} + \cdots + p_{ij})$th row of $\mathfrak{C}_i^{-1}$ with p_{ij} and $\mathfrak{C}_i$ being defined by (30). Then, control (29) with gain matrices

$$K_{s_i} = \begin{bmatrix} q_{i1}F_i^{p_{i1}-1}G_i \\ \vdots \\ q_{im}F_i^{p_{im}-1}G_i \end{bmatrix}^{-1} \begin{bmatrix} q_{i1}E_{i1}(F_i) \\ \vdots \\ q_{im}E_{im}(F_i) \end{bmatrix},$$
$$K_i = \begin{bmatrix} q_{i1}F_i^{p_{i1}-1}G_i \\ \vdots \\ q_{im}F_i^{p_{im}-1}G_i \end{bmatrix}^{-1} K'_i, \quad K'_i = B_{i2}^{-1}C_{i2}^{-1} \tag{32}$$

makes the system in (28) be a cooperative PS system.

Proof. It follows from Wang and Juang (1995) that pole placement formulas of K_{s_i} and K_i in (32) (except for that of K'_{ij}) place the closed loop poles at λ_{ij} (for $j = 1, \ldots, (n_i - m)$) and 0 (of geometric multiplicity 1). Hence, there is a similarity transformation T_i such that, under state transformation $x_i = T_i^{-1}z_i$, system (28) under control (29) becomes

$$\begin{cases} \begin{bmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \end{bmatrix} = \begin{bmatrix} A_{i,11} & 0 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} B_{i1} \\ B_{i2} \end{bmatrix} (K'_i u_i) \\ y_i = \begin{bmatrix} C_{i1} & C_{i2} \end{bmatrix} x_i, \end{cases} \tag{33}$$

where $A_i \triangleq T_i^{-1}(F_i - G_iK_{s_i})T_i$, $B_i \triangleq T_i^{-1}G_i$ with B_{i2} being square, $C_i \triangleq H_iT_i$, and $|\lambda_{ij}I - A_{i,11}| = 0$ for $j = 1, \ldots, (n_i - m)$. Since controllability and observability are invariant under similarity transformation, a direct computation of controllability and observability matrices of system (33) reveals that matrices B_{i2} and C_{i2} are invertible. Transformation of $w_{i1} = x_{i1}$ and $w_i = C_{i2}x_{i2}$ maps system (33) into the form of (12) with $A_{i,11}$ being Hurwitz, and the resulting system is a special case of (25). □

Example 4. Consider the nonminimum-phase system:

$$\dot{z}_{i1} = z_{i2}, \qquad \dot{z}_{i2} = v_i, \qquad y_i = -z_{i1} + z_{i2},$$

for which $y_i(s)/v_i(s) = (s-1)/s^2$. Under self-feedback control law $v_i = -2z_{i2} - u_i$, the individually closed-loop system becomes

$$\dot{z}_i = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} z_i + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u_i, \qquad y_i = \begin{bmatrix} -1 & 1 \end{bmatrix} z_i.$$

The transformation of $w_{i1} = z_{i2}$ and $w_{i2} = -(2z_{i1} + z_{i2})$ maps the system into

$$\dot{w}_i = \begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix} w_i + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u_i, \qquad y_i = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \end{bmatrix} w_i,$$

which is a cooperative PS system. ◇

The second design method of $u_{s_i}(z_i)$ is the backstepping design method (Krstic et al., 1995) which is applicable to the class of affine minimum-phase systems. Consider the nonlinear affine system

$$\dot{z}_i = F_i(z_i) + G_i(z_i)v_i, \qquad y_i = H_i(z_i). \tag{34}$$

The above system is said to have well-defined *relative degree* (Isidori, 1995; Khalil, 2003) of κ_i if the following set of conditions in terms of Lie derivatives and Lie brackets are met: $L_{G_i}L_{F_i}^j y_i = 0$ for $j = 0, \ldots, \kappa_i - 2$ and $[L_{G_i}L_{F_i}^{\kappa_i-1}y_i]^{-1}$ exist. Under this assumption, we can choose the following state transformation: $z'_{i1} = y_i$, $z'_{ij} = \frac{1}{a_{i0}}[L_{F_i}z'_{i(j-1)} + a_{i(j-1)}z'_{i(j-1)}]$ for $j = 2, \ldots, \kappa_i$, and $\xi_i = \xi_i(z_i) \in \Re^{n_i-\kappa_i m}$, where $a_{il} > 0$ for $l \in \{1, \ldots, \kappa_i\}$, $a_{i0} = (a_{i1} \cdots a_{i\kappa_i})^{1/\kappa_i}$, $z'_i = \text{vec}\{z'_{ij}\} \in \Re^{\kappa_i m}$ is the state of input–output dynamics, and ξ_i should be chosen such that the mapping from z_i to $[(z'_i)^T \ \xi_i^T]^T$ is diffeomorphic. Applying the transformation to (34) and choosing self-feedback control

$$v_i = [L_{G_i}L_{F_i}^{\kappa_i-1}y_i]^{-1}[-L_{F_i}z'_{i\kappa_i} - a_{i\kappa_i}z'_{i\kappa_i} + a_{i0}z'_{i1} + a_{i0}u_i], \tag{35}$$

yield the transformed system of

$$\begin{aligned} y_i &= z'_{i1} \\ \dot{z}'_{ij} &= -a_{ij}z'_{ij} + a_{i0}z'_{i(j+1)}, \quad j = 1, \ldots, \kappa_i - 1 \\ \dot{z}'_{\kappa_i} &= -a_{i\kappa_i}z'_{i\kappa_i} + a_{i0}z'_{i1} + a_{i0}u_i \\ \dot{\xi}_i &= \phi_i(\xi_i, z'_i), \end{aligned} \tag{36}$$

where $\phi_i(\cdot)$ represents the so-called internal dynamics. The ith system in (34) is said to be *minimum-phase* if the zero dynamics $\dot{\xi}_i = \phi_i(\xi_i, 0)$ is asymptotically stable. The following theorem provides the PS property and a canonical form for minimum-phase systems.

Theorem 4. *Suppose that the ith system in (34) is minimum-phase and of relative degree κ_i. Then, the resulting individually closed-loop system (36) under control (35) is a cooperative PS system, and it can be mapped into either (25) or the minimum-phase canonical form of*

$$\begin{bmatrix} \dot{w}'_{i1} \\ \dot{w}'_{i2} \end{bmatrix} = \begin{bmatrix} F'_{w'_{i1}}(w'_{i1}, w'_{i2}) \\ (A'_i \otimes I_m)w'_{i2} \end{bmatrix} + \begin{bmatrix} 0 \\ B'_i \otimes I_m \end{bmatrix} u_i$$
$$y_i = H'_i(w'_{i1}, w'_{i2}) + (C'_i \otimes I_m)w'_{i2} \tag{37}$$

where $B'_i = \begin{bmatrix} 0 & \cdots & 0 & a_{i\kappa_i} \end{bmatrix}^T$, $C'_i = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$, $H'_i(0, w'_{i2}) = 0$, and

$$A'_i = \begin{bmatrix} -a_{i1} & a_{i1} & 0 & \cdots & 0 \\ 0 & -a_{i2} & a_{i2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -a_{i(\kappa_i-1)} & a_{i(\kappa_i-1)} \\ a_{i\kappa_i} & 0 & \cdots & 0 & -a_{i\kappa_i} \end{bmatrix}.$$

Proof. It is straightforward to verify that, under the transformation of $w'_{i1} = \xi_i$ and $w'_{i2} = \text{vec}\{w'_{i2,j}\}$ with $w'_{i2,1} = z'_{i1}$ and $w'_{i2,j} = \frac{a_{i0}^{j-1}}{a_{i1}\cdots a_{i(j-1)}}z'_{ij}$ for $j = 2, \ldots, \kappa_i$, system (36) is mapped into (37) with $H'_i(\cdot, \cdot) \equiv 0$.

It is apparent that $-A'_i$ is the Laplacian of a ring digraph, hence matrix $-[A'_i + (A'_i)^T]$ is p.s.d. and of rank $(\kappa_i - 1)$. That is, matrix A'_i has $(\kappa_i - 1)$ eigenvalues in the left open half s-plane and one simple eigenvalue at the origin. It is straightforward to show that, under the transformation of $w_{i2} = \sum_{j=1}^{\kappa_i} \frac{w'_{i2,j}}{a_{ij}}$ and $w_{i1,j} = -w'_{i2,j} + \beta_i w_{i2}$ for $j = 1, \ldots, \kappa_i - 1$ and with $\beta_i = (a_{i1}^{-1} + \cdots + a_{i(\kappa_i-1)}^{-1})^{-1}$, the system of z'_i is transformed into (12). Incorporating internal dynamics into w_{i1} yields (25). □

4.2. High-level design of network-enabled controls

At the network level, the communication/sensing topology is specified by digraph $\{\mathcal{V}, \ \mathcal{E}\}$ or matrix S as in (2). Suppose without loss of generality that the Laplacian L (after permutation) is in the *lower triangular form*

$$L = \begin{bmatrix} L'_{11} & 0 & \cdots & 0 \\ L'_{12} & L'_{22} & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ \cdots & & & L'_{qq} \end{bmatrix} \tag{38}$$

where $L'_{ii} \in \Re^{l_i \times l_i}$ are irreducible. If L is irreducible, then $q = 1$ and $L'_{11} = L$. Graph $\{\mathcal{V}, \ \mathcal{E}\}$ is said to have at least *one global reachable node* (from which every other node can be reached) if and only if matrix L in (38) is *lower triangular complete* in the sense that for any $i \in \{2, \ldots, q\}$, L'_{ij} with $j < i$ are not all identically zero.

A network-enabled distributed control must conform with the information structure S as defined in (2). In the proposed network-level design, the distributed control is simply chosen to be as in (13) which is known to work for the *fictitious integrator systems*

$$\dot{y}_i = u_i, \quad i = 1, \ldots, n_s. \tag{39}$$

Our goal is to show that the control in (13) also works for heterogeneous systems. To this end, the concept of PS systems has been used to quantify the impact of heterogeneous dynamics on their networked operation, and two designs of self-feedback controls v_{s_i} have been presented to make the heterogeneous systems become cooperative PS systems. In the next subsection, it is shown, using the impact equivalence principle (which includes Lemma 1 and Theorem 2 as special cases), that these controls separately designed work well in any networked operation and that a non-trivial global consensus emerges if and only if Laplacian L is lower triangularly complete.

4.3. Impact equivalence principle

The *impact equivalence principle*, stated as Theorem 5, simplifies the design and analysis of cooperative networked systems by modularizing the designs of individual controls as well as the network-level controls. In particular, the networked control of cooperative PS systems does not require any explicit information about the heterogeneous physical systems except that their impact coefficients are not larger than a design threshold $\overline{\epsilon}$. Hence, heterogeneous systems can be switched into and out of service at any node in the overall network. This plug-and-play feature of networked operations could be advantageous in many applications.

Theorem 5. *Suppose that self-feedback controls $v_{s_i}(\cdot)$ are designed (as was done in Section 4.1) such that the resulting systems become cooperative PS systems (as defined in Definition 3). Then, the distributed control in (13) always ensures either local or global (non-trivial) consensus of their outputs provided that the gains k_{y_i} are chosen according to*

$$0 < k_{y_i} < \min_{i \in \{1,2,\ldots,q\}} \frac{\lambda'(\Gamma_i L'_{ii} + (L'_{ii})^T \Gamma_1)}{\overline{\epsilon}\lambda_{\max}((L'_{ii})^T \Gamma_i L'_{ii})}, \tag{40}$$

where L'_{ii} are as defined in (38), Γ_1 is determined based on L'_{11} as was done in Lemma 1, Γ_i with $i \geq 2$ is determined based on L'_{ii} as solved in Theorem 2, and $\lambda'(A)$ denotes the smallest nonzero eigenvalue of matrix A. Furthermore, a global (non-trivial) consensus (of $\lim_{t\to+\infty} y_i(t) = c$ for all i) is ensured if and only if graph $\{\mathcal{V}, \ \mathcal{E}\}$ has at least one global reachable node.

Proof. For all the systems in the network, let $\mathcal{N}_\mu$ be their index sets corresponding to blocks of $L'_{\mu\mu}$ in (38). The proof is inductive with respect to $\mu \in \{2, \ldots, q\}$ as in the following steps:

Step 1 ($\mu = 1$): It follows from Lemma 1 that $z_i \in L_\infty$, $(y_i - y_j) \in L_2$, $u_i \in L_2$ and $\lim_{t\to+\infty} y_i(t) = c_1$ for all $i, j \in \mathcal{N}_1$ and for some $c_1 \in \Re^m$. To find c_1, we know from the first left eigenvector γ_1 of L'_{11} that $\gamma_1^T L'_{11} = 0$ and in turn $[(\gamma_1^T K_{h1}^{-1}) \otimes I_m] u_{\mathcal{N}_1} \equiv 0$, where $K_{h1} = \mathrm{diag}\{k_{y_i} : \ i \in \mathcal{N}_1\}$. It follows from $\dot{w}_{i2} = u_i$ in (25) that

$$\sum_{i\in\mathcal{N}_1} \frac{\gamma_{1_i}}{k_{y_i}} w_{i2} \equiv c_1 \sum_{i\in\mathcal{N}_1} \frac{\gamma_{1_i}}{\beta_i k_{y_i}} \Longrightarrow c_1 \triangleq \frac{\sum_{i\in\mathcal{N}_1} \frac{\gamma_{1_i}}{k_{y_i}} w_{i2}(t_0)}{\sum_{i\in\mathcal{N}_1} \frac{\gamma_{1_i}}{\beta_i k_{y_i}}}. \tag{41}$$

Using the output equation of (25) and applying Chebyshev inequality and Barbalat lemma, we can verify that $(y_j - c_1) \in L_2$ and hence $\lim_{t\to+\infty} y_i(t) = c_1$ for any $j \in \mathcal{N}_1$ since

$$\begin{aligned} &\|y_j - c_1\|^2 \left(\sum_{i\in\mathcal{N}_1} \frac{\gamma_{1_i}}{\beta_i k_{y_i}} \right)^2 \\ &\quad = \left\| \sum_{i\in\mathcal{N}_1} \frac{\gamma_{1_i}}{\beta_i k_{y_i}} [(y_j - y_i) + (y_i - \beta_i w_{i2})] \right\|^2 \\ &\quad \leq 2m_{\mathcal{N}_1} \sum_{i\in\mathcal{N}_1} \frac{\gamma_{1_i}^2}{\beta_i^2 k_{y_i}^2} [\|y_j - y_i\|^2 + \|H_{w_i}\|^2], \end{aligned}$$

where $m_{\mathcal{N}_1}$ is the number of entries in index set $\mathcal{N}_1$.

Step 2 ($\mu = 2$): If $L'_{21} = 0$, we can repeat step 1 to show that $\lim_{t\to+\infty} y_i(t) = c_2$ for all $i \in \mathcal{N}_2$, where $c_2 \in \Re^m$ is a local consensus (among the systems in set $\mathcal{N}_2$) and can similarly be found as was done in (41) (but generally $c_2 \neq c_1$). If $L'_{21} \neq 0$, all the nodes in $\mathcal{N}_2$ can be reached from any node in $\mathcal{N}_1$, and it follows from Theorem 2 that $z_i \in L_\infty$, $(y_i - y_j) \in L_2$, $u_i \in L_2$ and $\lim_{t\to+\infty} y_i(t) = c_1$ for all $i, j \in \mathcal{N}_2$.

Step p (for $p = 3, \ldots, q$): Suppose that the result holds for $\mu = 1$ up to $\mu = p - 1$. Then, the result of $\mu = p$ can be concluded in a similar fashion as that of $\mu = 2$. □

5. Modularized design: varying topologies

In many applications, networked controls need to be implemented without the restriction of sensing and communication topologies being fixed. To this end, let $\aleph$ be the set of positive integers and let $\{t_k : k \in \aleph\}$ be the time sequence at which the communication/sensing matrix $S(t)$ defined in (2) experiences a change. Then, for any sub-sequence $\{k_\eta : \eta \in \aleph\}$, the composite graph $(\mathcal{V}, \mathcal{E}^c(\eta))$ corresponds to the composite matrix

$$S^c(\eta) \triangleq S(t_{k_{\eta+1}-1}) \circ S(t_{k_{\eta+1}-2}) \circ \cdots \circ S(t_{k_\eta}), \tag{42}$$

where $\circ$ denotes the Hadamard product. The following definition prescribes the cumulative information flow, and the subsequent theorem provides a class of PS systems that are plug-and-play ready and can successfully be operated with an intermittent information network.

Definition 4. A varying graph $(\mathcal{V}, \mathcal{E}(t))$ is said to be cumulatively connected if $(t_{k_{\eta+1}} - t_{k_\eta}) \leq T$ for some constant $T \geq 0$ and if, for every η, the composite graph $(\mathcal{V}, \mathcal{E}^c(\eta))$ has at least one globally reachable node or, equivalently, the composite matrix $S^c(\eta)$ is lower triangular complete.

Theorem 6. *Suppose that control $v_{s_i}(\cdot)$ in (3) is designed so that, under a diffeomorphic transformation $z_i = \mathcal{Z}'_i(w'_i)$, the systems in the form described in (6) can be transformed into (37) in which $w'_{i1} \in L_\infty$ and $\|H'_i(w'_{i1}, w'_{i2})\| \leq \alpha_{i1}(w'_i(t_0))e^{-\alpha_{i2}(t-t_0)}$ for some $\alpha_{i1}(w'_i(t_0)) \geq 0$ and $\alpha_{i2} > 0$. Consider the distributed control (13) with $k_{y_i} \in (0, a_{ik_i}/n_s]$ and with $S(t)$ and $L(t)$ being time varying. Then, a nontrivial consensus of $\lim_{t\to+\infty} y_i(t) = c$ can be ensured if and only if the varying graphs are cumulatively connected.*

Proof. It follows from (13) and (37) that

$$u_i = k_{y_i} \sum_{j=1}^{n_s} S_{ij}(t)[(C_j' \otimes I_m)w_{j2} - (C_i' \otimes I_m)w_{i2}) + \delta_i(t),$$

where $\delta_i(t) = k_{y_i} \sum_{j=1}^{n_s} S_{ij}(t)[H_j'(w_{j1}', w_{j2}') - H_i'(w_{i1}', w_{i2}')]$. Defining $x = \text{vec}\{w_{i2}\}$ and $\delta = \text{vec}\{\delta_i(t)\}$, we have

$$\dot{x} = -\left[\overline{L}'(t) \otimes I_m\right] x + [B' \otimes I_m]\delta, \quad (43)$$

where $B' = \text{diag}\{B_i'\}$, $D(t) = \text{diag}\{S(t)\mathbf{1}\}$, $L(t) = D(t) - S(t)$, and $\overline{L}'(t) = [\overline{L}_{ij}'(t)]$ with

$$\overline{L}_{ij}'(t) = \begin{cases} -A_i' + L_{ii}(t)B_i'C_i' & \text{if } i = j \\ L_{ij}(t)B_i'C_i' & \text{if } i \neq j. \end{cases}$$

It follows from Lemma 5.5 in Qu (2009) that Laplacian $\overline{L}'$ has the same topological property as that of $L(t)$. System (43) can simply be viewed as a piecewise-constant linear system with diminishing uncertainty $\delta(t)$, and hence its solution is

$$x(t_{k+1}) = P(k)x(t_k) + \delta'(k), \quad (44)$$

where $P(k) = e^{-\left[\overline{L}'(t_k)\otimes I_m\right](t_{k+1}-t_k)}$ is nonnegative and row-stochastic, and

$$\begin{aligned} |\delta_i'(k)| &= \left| \int_{t_k}^{t_{k+1}} \left[e^{-\left[\overline{L}'(t_k)\otimes I_m\right](t-\tau)} B'\delta(\tau) \right]_i d\tau \right| \\ &\leq \int_{t_k}^{t_{k+1}} \max_j |\delta_j(\tau)| d\tau \\ &= \frac{\max_j \alpha_{j1}'}{\min_j \alpha_{j2}'} \left(e^{-\min_j \alpha_{j2}' t_k} - e^{-\min_j \alpha_{j2}' t_{k+1}} \right). \end{aligned}$$

It is known (e.g., Theorem 4.53 in Qu, 2009) that, if $\delta'(k) = 0$, $x(t_k)$ reaches consensus as k approaches infinity if and only if varying graphs are cumulatively connected. Since the nonnegative series

$$\sum_{k=0}^{\infty} \max_i |\delta_i'(k)| \leq \frac{\max_j \alpha_{j1}'}{\min_j \alpha_{j2}'} e^{-(\min_j \alpha_{j2}')t_0}$$

is convergent and therefore is a Cauchy sequence, we conclude from Lemma 5.29 in Qu (2009) that system (44) reaches consensus if it does when $\delta'(k) = 0$. □

6. Conclusions

In this paper, an input feedforward passivity index is used to characterize the input–output relationship for heterogeneous dynamical systems of potentially high-relative degrees and/or nonminimum-phase. It is shown that self-feedback controls can individually be designed to make heterogeneous systems ready for networked operations, that a network-enabled distributed control can be designed independently of specific dynamics, and that these controls separately designed can together ensure a global (nontrivial) consensus under minimum information flows. Both fixed and varying topologies of the local information network are considered.

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Appendix. Proofs

Proof of Proposition 1. (i) is obvious since (8) is equivalent to $\Phi_i(z_i, u_i) = u_i^T y_i^a - \eta_i(z_i)$.

(ii) It follows from (9) and (10) that

$$\begin{aligned} \dot{V}_i &= \left(\frac{\partial V_i}{\partial z_i}\right)^T \mathcal{F}_i^c(z_i, u_i) \\ &\leq -\gamma_{i3}\|z_i\|^2 + \left\|\frac{\partial V_i}{\partial z_i}\right\| \cdot \|\mathcal{F}_i^c(z_i, u_i) - \mathcal{F}_i^c(z_i, 0)\| \\ &\leq -\gamma_{i3}\|z_i\|^2 + \gamma_{f_i}\gamma_{i4}\|z_i\| \, \|u_i\| + [\|y_i\| \cdot \|u_i\| + u_i^T y_i] \\ &\leq -\gamma_{i3}\|z_i\|^2 + (\gamma_{f_i}\gamma_{i4} + \gamma_{h_i})\|z_i\| \, \|u_i\| + u_i^T y_i, \end{aligned}$$

from which inequality (7) can always be established for $\epsilon_i \geq (\gamma_{f_i}\gamma_{i4} + \gamma_{h_i})^2/(2\gamma_{i3})$. □

Proof of Proposition 2. It follows from (6) that

$$\dot{V}_i = \mathcal{L}_{F_i^c} V_i + (\mathcal{L}_{G_i} V_i)u_i = -\eta_i'(z_i, u_i) + u_i^T y_i + \frac{\epsilon_i}{2}\|u_i\|^2,$$

where $\eta_i'(z_i, u_i) \triangleq -\mathcal{L}_{F_i^c} V_i - (\mathcal{L}_{G_i} V_i - H_i^T)u_i + \frac{\epsilon_i}{2}\|u_i\|^2$. Hence, the system is passivity-short if and only if, for all $u_i \in \Re^m$, function $\eta_i'(z_i, u_i)$ is p.s.d. The proof is completed by noting that $\eta_i'(z_i, u_i) \geq \eta_i(z_i)$ and that, when $u_i = (\mathcal{L}_{G_i} V_i - H_i^T)^T/\epsilon_i$, $\eta_i'(z_i, u_i) = \eta_i(z_i)$. □

Proof of Proposition 3. Since $F_{i,11}$ is Hurwitz, matrix solution $P_{i,11}$ to Lyapunov equation $P_{i,11}A_{i,11} + A_{i,11}^T P_{i,11} + I = 0$ is p.d. Choosing storage function $V_i = \frac{1}{2}z_{i1}^T P_{i,11} z_{i1} + \frac{1}{2}z_{i2}^T z_{i2}$, we know that the system is PS because

$$\begin{aligned} \dot{V}_i &= -\|z_{i1}\|^2 + z_{i1}^T[P_{i,11}G_{i1} - H_{i1}]u_i + y_i^T u_i \\ &\leq \frac{\epsilon_i}{2}\|u_i\|^2 + y_i^T u_i, \end{aligned}$$

where $\epsilon_i \geq \|P_{i,11}G_{i1} - H_{i1}\|^2/2$. □

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